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ILSR rumor spreading model with degree in complex network

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HIGHLIGHTS

- We establish a differential equation of rumor propagation dynamics.
- A new node-state transition function is proposed to describe rumor propagation.
- Combining the actual social network, a new rumor propagation model is proposed.
- The actual influence of different degrees and network topology is investigated.

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ABSTRACT

Most rumors in social networks are extremely harmful and have a significant negative impact on social welfare. Therefore, exploring the laws of rumor propagation has been one of the hot topics in current researches. Most traditional rumor spreading models are based on infectious disease transmission models, such as SIR. Since the influence of individual differences and the network structure on rumor spreading are not considered, the rumor propagation process in complex networks can only be described in a coarse-grained manner. In this paper, we consider the role of different users in rumor propagation. Based on the degree of different nodes in the network, we design a new state transition function for each node and proposed a new rumor propagation ILSR model. Firstly, we analyze the model, calculate the equilibrium point and the basic reproductive number to prove the rationality of the model. Then experiments are performed in WS networks, BA scale-free networks and a real Facebook network to investigate the relationship between various nodes with time and the impact of network structure on rumor propagation, and the experimental results show the correctness and effectiveness of the model. It provides a reference for exploring the propagation law of rumors in complex networks and guiding and controlling the propagation of rumors.

1. Introduction

Rumor usually refers to the unconfirmed elaboration of public concerns, issues and events related to public interests by various means of dissemination [1]. With the rapid development of the Internet, the Online Social Network (OSN) has penetrated into every aspect of people's production and life. However, a series of rumors that it breeds are eroding people's daily lives, impacting the normal life of the Internet and society. Compared with rumors in daily society, rumors in cyberspace spread faster, have a wider range of influence, and have more uncontrollable factors. Therefore, the study of rumor propagation laws in complex networks represented by social networks has positive significance for the control of rumor [2].

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Most of the traditional models for analyzing information transmission are based on dynamics models of infectious diseases, such as SI (Susceptible–Infected), SIS (Susceptible–Infected–Susceptible), SIR (Susceptible–Infected–Removed) and so on. Researchers in different fields have made corresponding improvements to these models based on their research scenarios. For instance, Witbooi et al. [3] proposed a random SEIR epidemiological model that demonstrates the exponential stability theorem that almost determines disease-free balance. Liu et al. [4] proposed the standard incidence of random SIRS epidemiological models, [5–8] based on SIS, SIR compartmental model to study the spread of epidemics. These results indicate the effectiveness of the SIR compartmental model in epidemiological research, and provide a rich theoretical basis for alleviating the epidemic of infectious diseases and predicting the development trend of infectious diseases. The spread of online rumors is similar to the spread of infectious diseases. Therefore, the infectious disease dynamics model is also widely used in rumor propagation analysis on social networks, such as propagation modeling of rumors, tracing of information, etc. Wan et al. [9] presented an improved rumor propagation model, defined as the Spreader–Ignorant–Eliminate–Rstifler–Estifler (SIERsEs) model. Based on the microscopic Markov chain approach, Zheng et al. [10] build the probability tree to describe the switching process between different states, and Li et al. [11] proposed a model of SIQR for the problem of invalid isolation of virus or rumor propagation. The exponential separation of rumor propagation time between two variants is studied, and the propagation law of rumors between informed and uninformed users is analyzed [12]. However, most of the model studies of analytic rumors are based on uniform networks. A large number of studies have demonstrated that social networks in reality are complex networks with small worlds and scale-free features [13–15]. The mean-field approach based on the SIR model over-simplifies the process of rumor propagation, so it can only describe the process of rumor propagation of online social networks in a coarse-grained manner. SIR model assumes that the propagation probability among network nodes is equal in unit time, and the characteristics of social networks are not fully considered, so it is inconsistent with the propagation characteristics of actual social networks [16–18]. In order to better understand the mechanism of rumor propagation, it is helpful to study the rumor propagation and explore the model and structure in complex networks. Sun et al. [19] considered real-world rumor propagation characteristics and influencing factors, and introduced rumor acceptable functions to describe the propagation rates of distinct nodes. Singh et al. [20] considered that the correlation between degree and degree has a considerable influence on the rumor propagation in real-world networks, and the propagation law of rumors is studied in the scale-free network. Considering that exposed nodes may become removed nodes at a certain rate, Liu et al. [21] analyzed the SEIR rumor propagation model in heterogeneous networks. The impact of community size heterogeneity and the intensity of community structure on the dynamic behavior of rumor communication has also been analyzed [22]. Because individuals in complex networks have different understandings of specific rumors, individual characteristics are also taken into account [23,24]. Network rumors are deeply analyzed in social networks represented by small world networks and scale-free networks, which indicate that the degree distribution, heterogeneity and distance distribution of nodes have different effects on rumor propagation [25,26].

Previous studies have conducted in-depth analysis of rumors in complex networks, but different users have different levels of rational knowledge in social networks, and there are differences in their understanding of specific rumor fields. Therefore, considering such differences, their role in the spread of rumors is different, and the rates of transmission and recovery of rumors are also different [23]. Based on the above discussion, we propose a new rumor spreading ILSR model, and combine the existing research methods for improving the SIR model to analyze the model. Further, considering the complexity of the actual social network structure, in the complex network, we redefined the conversion functions of transmission rate, infection rate and recovery rate for each node based on the different degrees of nodes to make ILSR model more reasonable and effective. Finally, simulation experiments are carried out in WS small world networks, BA scale-free networks and an actual Facebook social network, which verify the rationality of the model and reveal the network topology and the actual impact of different nodes on rumor propagation.

The paper is organized as follows. In Section 2, we introduce some traditional models based on SIR model to study rumor propagation, and explain the shortcomings of the model for research rumor in the complex network. In Section 3, we present a new rumor spreading ILSR model and introduce different propagation mechanisms of the model in homogeneous network and complex network. In Section 4, simulation experiments in the regular network, SW small world network, BA scale-free network and a real network are performed to test and verify the analysis results. Finally, conclusions are given in Section 5.

2. Traditional model

Cyberspace rumors emerge in endlessly, and their influence is becoming ever more serious. Facing this phenomenon, the majority of scholars began to conduct detailed modeling research on the propagation law of rumors. Most current rumor propagation models are built on infectious disease models.

SIR is a classical model of infectious disease, which divides the population into three groups, S(Susceptible), I(Infected), and R(Removed). If an individual has effective contact with other individuals in unit time (enough contact to cause virus transmission, i.e. Susceptible and Infected), the probability of transmitting the virus is β, and the probability of the individual being cured within unit time is γ. The SIR(Susceptible–Infected–Removed) model shown in Fig. 1 can be established.

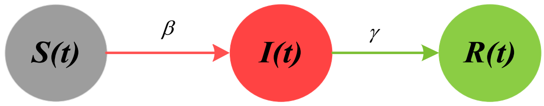

Fig. 1. Infectious disease SIR model.

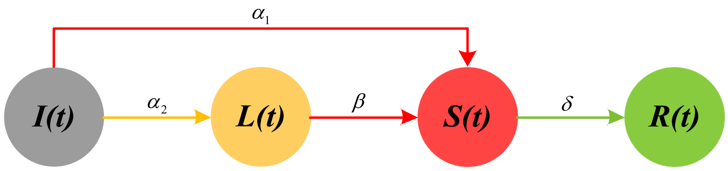

Fig. 2. ILSR rumor propagation model.

The mean-field equations of the SIR model are defined as follow:

$$\begin{cases} \dfrac{dS(t)}{dt} = -\beta S(t)I(t) \\ \dfrac{dI(t)}{dt} = \beta S(t)I(t) - \gamma I(t) \\ \dfrac{dR(t)}{dt} = \gamma I(t) \end{cases} \tag{1}$$

The SIR(Susceptible–Infected–Removed) model is suitable for infectious diseases that are lifelong immunity after rehabilitation. Although the model is simple, it has gained very valuable concepts and conclusions. It introduces a parameter $\sigma = \beta/\gamma$ to indicate the average number of contacts that the infected person has during the infection. If $\sigma_{S0} \leq 1$, when $t \to \infty$, $i(t)$ decreases to 0, indicating that the infectious disease has disappeared. If $\sigma_{S0} > 1$, $i(t)$ first increases to the maximum peak and then gradually decreases to zero. It can be observed that the final state of the SIR model is non-infected, indicating that there is no susceptible person and infection, all of them are recovered, and the infectious diseases eventually disappeared.

It has been used by many researchers to study the process of information dissemination in social networks. A person who is easy to spread information is affected by a spreader and then becomes a spreader. For some reason, the spreader stops spreading the information and becomes a removal. The conversion parameters β and γ in the process are statistically based probabilities, and the state conversion rates of all users are the same, which does not combine the influence of individual differences on information dissemination. Although researchers have considered different factors to improve the SIR model, they hope to establish a more reasonable compartment model (such as SEIR model, Susceptible–Exposed–Infected–Removed) to describe the spread of rumors in the network. However, due to the complexity of social network topology and the numerous factors affecting information transmission, those models are unable to describe the way of information transmission in social networks, which are far from the actual situation [22,23].

3. ILSR model

3.1. Model description

In order to better describe the law of rumor propagation in model social networks, we propose an ILSR (Ignorant–Lurker–Spreader–Removal) rumor propagation model. In the ILSR model, we divide the people in the social network into four groups: I (Ignorant, who has never been exposed to rumors, and may believe rumors), L (Lurker, who has heard the rumors, but Skeptical, temporarily not spreading rumors), S (Spreader, who believe in rumors, and spread rumors in the network with a certain probability), R (Removal, who identify this information as false information or lose interest in rumors, no longer participate the spread of rumors). $I(t)$, $L(t)$, $S(t)$ and $R(t)$ respectively represent the proportion of the population in these four groups at time t, then

$$I(t) + L(t) + S(t) + R(t) = 1 \tag{2}$$

The transition process of the four states is shown as Fig. 2.

As shown in Fig. 2, the laws of the ILSR model and their expressions can be summarized as follows: (a) When an ignorant contacts with a spreader, the ignorant will become a lurker with probability α_1 or a spreader with probability α_2 due to the user's own situation. The lurkers do not spread rumors, and the spreaders will spread rumors. (b) when a lurker contacts a spreader, the lurker will become the spreader with probability β and begin to spread rumors.

(c) when a spreader contacts a removal, the spreader will become the removal with probability δ and stop to spread rumors.
(d) the total number of users is still kept constant regardless of factors such as increase, decrease, and flow of users.

According to the node state transition process described in Fig. 2, the following differential dynamics equation of rumor propagation can be established.

$$\begin{cases} \dfrac{dI(t)}{dt} = -\alpha_1 I(t)S(t) - \alpha_2 I(t)S(t) \\ \dfrac{dL(t)}{dt} = \alpha_2 I(t)S(t) - \beta L(t) \\ \dfrac{dS(t)}{dt} = \alpha_1 I(t)S(t) + \beta(t)L(t) - \delta S(t) \\ \dfrac{dR(t)}{dt} = \delta S(t) \end{cases} \tag{3}$$

The initial condition of Eq. (3) is $\{(I, L, S, R)\,|I, L, S, R \geq 0 \text{ and } I + L + S + R = 1\}$.

3.2. Model analysis in homogeneous network

The ILSR model of Eq. (3) is analyzed, and the equilibrium point of the model is calculated to prove the validity of the model.

$$\begin{cases} -\alpha_1 I(t)S(t) - \alpha_2 I(t)S(t) = 0 \\ \alpha_2 I(t)S(t) - \beta L(t) = 0 \\ \alpha_1 I(t)S(t) + \beta L(t) - \delta S(t) = 0 \\ \delta S(t) = 0 \end{cases} \tag{4}$$

Let $x = (I, L, S, R)^T$, then, the system (3) can be written as $x' = F(X) - V(X)$, where

$$F(x) = \begin{bmatrix} \alpha_2 IS \\ \alpha_1 IS \\ 0 \\ 0 \end{bmatrix}, \quad V(x) = \begin{bmatrix} \beta L \\ -\beta L + \delta S \\ \alpha_1 IS + \alpha_2 IS \\ -\delta S \end{bmatrix} \tag{5}$$

The Jacobian matrices of $F(x)$ and $V(x)$ at the non-toxic equilibrium point P_0 are

$$F = DF(P_0) = \begin{bmatrix} 0 & \alpha_2 I^* \\ 0 & \alpha_1 I^* \end{bmatrix} \tag{6}$$

$$V = DV(P_0) = \begin{bmatrix} \beta & 0 \\ -\beta & \delta \end{bmatrix} \tag{7}$$

then

$$FV^{-1} = \begin{bmatrix} 0 & \alpha_2 I^* \\ 0 & \alpha_1 I^* \end{bmatrix} * \begin{bmatrix} \dfrac{\delta}{\beta} & 0 \\ \dfrac{1}{\delta} & \dfrac{1}{\delta} \end{bmatrix} = \begin{bmatrix} \dfrac{\alpha_2 I^*}{\delta} & \dfrac{\alpha_2 I^*}{\delta} \\ \dfrac{\alpha_1 I^*}{\delta} & \dfrac{\alpha_1 I^*}{\delta} \end{bmatrix} \tag{8}$$

Let $\left|FV^{-1} - \lambda E\right| = 0$, then

$$\left|FV^{-1} - \lambda E\right| = \begin{vmatrix} \dfrac{\alpha_2 I^*}{\delta} - \lambda & \dfrac{\alpha_2 I^*}{\delta} \\ \dfrac{\alpha_1 I^*}{\delta} & \dfrac{\alpha_1 I^*}{\delta} - \lambda \end{vmatrix} = 0 \tag{9}$$

and there

$$R_0 = \rho(FV^{-1}) = \frac{\alpha_1 + \alpha_2}{\delta} I^* \tag{10}$$

When $R_0 < 1$, there is no rumor after the system is stable, on the contrary, there are still rumors in the system.

In order to verify the correctness of the conclusions obtained, ILSR model was numerically simulated.

Fig. 3 shows the trend of the proportion of the four groups in the ILSR model over time. The initial condition is that there is only one rumor spreader in the network, so $S(0) = 0.0001$, $I(0) = 0.9999$, $L(0) = 0$ and $R(0) = 0$. As can be viewed, when the spreader starts to spread rumors, its number will increase rapidly. With the further spread of rumors, the number of spreaders reaches the maximum peak, then decreases, and finally the number of spreader is zero, which indicates that the rumor stops spreading. The number of lurkers also increases to a peak, then decreases and finally

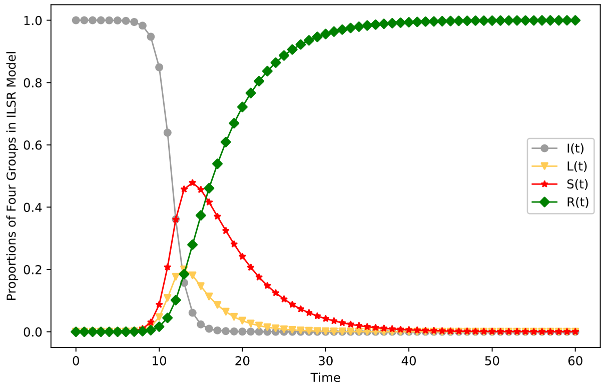

Fig. 3. Proportions of Ignorant(I), Lurker(L), Spreader(S), Removal(R) over time with $\alpha_1 = 0.6$, $\alpha_2 = 0.4$, $\beta = 0.15$, $\delta = 0.1$.

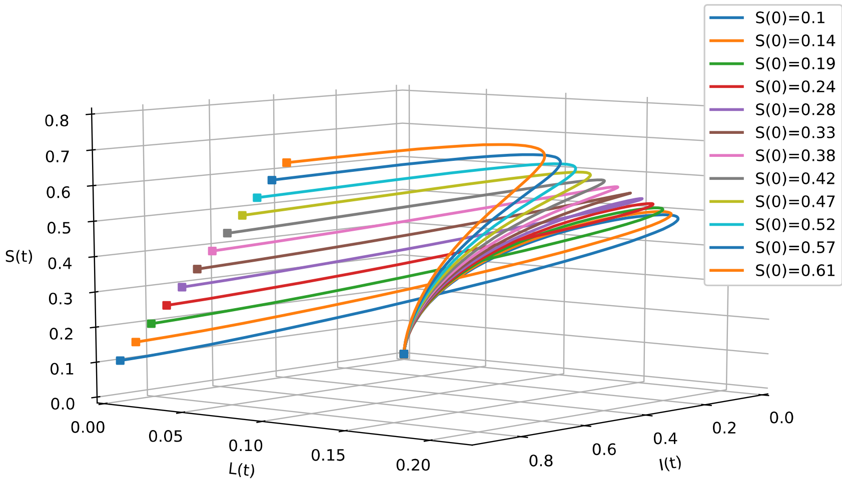

Fig. 4. Some solutions of the model with certain given parameters. $\alpha_1 = 0.6$, $\alpha_2 = 0.4$, $\beta = 0.15$, $\delta = 0.1$.

disappears. In the end, only removals exist in this model. Because the propagation threshold parameter $R_0 = 0$ in this initial condition, the system no longer has rumors, which show the correctness of our analysis results.

Fig. 4 shows the influence of different initial $S(0)$ on rumor propagation results. The initial condition is that the proportion of rumor spreaders $S(0)$ ranges from 0.10 to 0.61, $I(0) = 1 - S(0)$, $L(0) = 0$ and $R(0) = 0$. It can be seen that after the system is stable, the value of $I(t)$, $L(t)$ and $S(t)$ eventually converges to $(I, L, S) = (0, 0, 0)$, which indicates that the number of lurkers and spreaders is zero. Therefore, different initial conditions do not affect the number of spreaders after the system stabilizes. There will be no lurkers and spreaders in the system, and the rumor will eventually disappear.

3.3. Model analysis in complex networks

In the first two subsections, we analyzed the ILSR model, and the results of the simulation experiments show that the rumors will eventually disappear. However, in the real social network, the complex network topology will affect the spread of rumors. In addition, different individuals have different roles in the rumor propagation process, and the influence of individual differences on rumor propagation should be considered. When a person hears a rumor, based on his own understanding of the existing knowledge, he may spread the rumor, or he may not spread it, and remain lurking. If he chooses to be a lurker, when there are many people around him spreading the rumor, he will become a spreader with a greater probability to spread the rumor. Similarly, depended upon their own subjective judgments, the probability that different people stop spreading rumors is different. Therefore, we divide users into important users (rich knowledge, strong immunity to rumors, better recognition of rumors, generally do not easily disseminate information) and ordinary users (relatively lacking knowledge, easy to blindly believe others). Based on the above discussion, we re-analyze ILSR rumor propagation model in a complex network. Real people are regarded as nodes in the network, and the connection between users is regarded as the edges of nodes. Based on the degree of each node, a new node state transition function is designed to make the model more reasonable.

The average degree of the network is $\bar{k}$. If the degree of node i is greater than the average degree $\bar{k}$, node i is considered as an important node, otherwise, it is an ordinary node. When a normal node i is affected by the spreader j, the node i changes from the ignorant to a lurker or a spreader is determined by the relationship between its degree k_i and the average degree $\bar{k}$. If $k_i > \bar{k}$, node i becomes a lurker, the lurker does not spread rumors, and conversely, node i becomes a spreader, and begins to spread rumors. In addition, an important spreader becomes a removal with a greater probability than a normal node.

Therefore, the state transition function of each node is defined as follows:

In a complex network, the degree set of nodes is

$$k_{\Omega} = \{k_1, k_2, k_3, \ldots, k_i, k_j, \ldots, k_n\} \tag{11}$$

so the average of the nodes is

$$\bar{k} = \frac{\sum_{i\in\Omega} k_i}{N} \tag{12}$$

where, $\bar{k}$ is the average degree, Ω is the node set of the network, and N is the total number of nodes.

A spreader node i spreads rumors to a ignorant node j, and the probability that node j becomes a lurker or a spreader is defined as:

$$\alpha_{f(k_j)}(i,j) = 1 - \frac{k_j}{\sum_{r\in\tau(i)} k_r} \tag{13}$$

$$f(k_j) = \begin{cases} 1, & k_j \le \bar{k} \\ 2, & k_j > \bar{k} \end{cases} \tag{14}$$

where, $\tau(i)$ represents the neighbor node set of node i, and k_j represents the degree of node j. According to the degree of affected node j, the values of conversion rates α_1 and α_2 can be calculated. It can be concluded that the probability of spreading rumors for each node is different. Compared to the homogeneous network, individual differences are considered, which is more consistent with the actual situation of social networks.

After a lurker j is affected by a spreader i, the probability that its state changes from a lurker to a spreader is

$$\beta(i,j) = \frac{k_i}{k_i + k_j} \tag{15}$$

It can be seen that it is more difficult for ordinary nodes to spread rumors to important nodes, and vice versa.

The probability that a spreader i becomes a removal is defined as

$$\delta(i) = \frac{k_i + \sum_{r\in\zeta(i)} k_r}{k_i + \sum_{j\in\tau(i)} k_j} \tag{16}$$

where, $\tau(i)$ is the set of neighbor nodes of node i, and $\xi(i)$ represents the node set of removals in the neighbor of node j. It can be seen that the recovery rate of the rumor spreader i is related to both itself and the neighbors. The greater the degree of node i, the more removals in the neighbor nodes, the more likely it becomes a removal to stop spreading rumors.

The above transition function can be illustrated by a simple example in Fig. 5.

As shown in Fig. 5, the total node of the network is 11 and the average degree is 3. there are four groups of all nodes, including I, L, S and R. The rumor propagation process includes nine stages. At each time, the state transition of the node is as follows:

t1: Node 1 is a rumor spreader in the network, spreads rumors to neighbor nodes 2, 3, 4 and 6. According to Eq. (13), the propagation probability of each node can be calculated, $\alpha_2(1,2) = 11/18$, $\alpha_1(1,3) = 15/18$. Because node 2 has a larger degree, it has a lower probability to accept and spread rumors.

t2: Node 1 successfully spreads rumors to nodes 3, 4 and 6 with different probabilities. Because the degree of these nodes is less than the average degree $\bar{k}$ (see Eq. (14)), they all become spreaders.

t3: Nodes 2 and 7 are affected by the spreader. Because $k_2 > \bar{k}$, node 2 becomes a lurker and does not spread rumors. On the contrary, $k_7 < \bar{k}$, node 7 becomes a spreader and begin to spread rumors.

t4: Node 2 is affected by multiple nodes. According to Eq. (15), $\beta(1,2) = 5/12$, $\beta(3,2) = 3/10$, $\beta(6,2) = 3/10$, $\beta(7,2) = 2/9$. Finally, node 2 is converted from a lurker to a spreader.

t5: Nodes 8 and 9 are both converted to spreaders, while node 5 is converted from a spreader to a removal by probability $\delta(5) = 3/13$ (see Eq. (16)).

t6: Node 10 is converted to a spreader. Meanwhile, nodes 4 and 6 are converted to removals with probabilities $\delta(4) = 5/10$ and $\delta(6) = 6/11$ respectively.

t7–t9: At these moments, the probability of node transition is calculated in a similar way. Finally, all nodes in the process become removals and stop spreading rumors.

However, the network topology will affect the spread of rumors, so rumors spread in the network has many different results. For example, at time t3 and time t6, rumor spreading may appear as showed in Fig. 6.

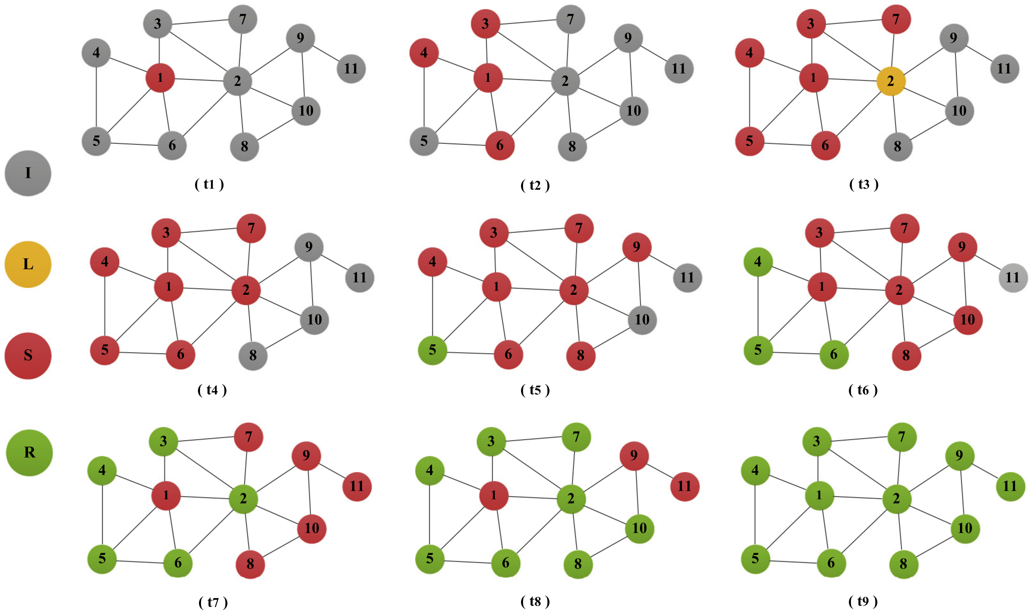

Fig. 5. An example of a rumor spreading process. $N = 11$, $\bar{k} = 3.09$.

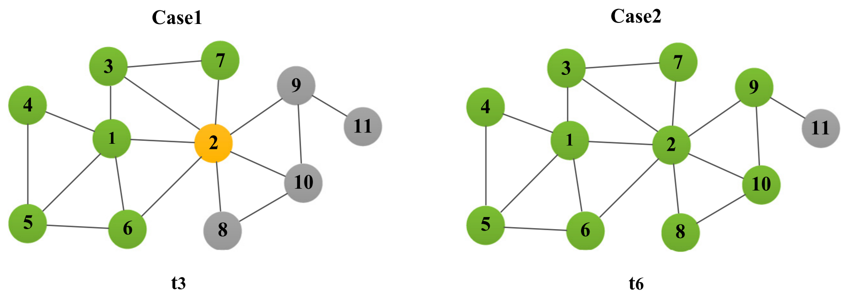

Fig. 6. Other results of rumor spread at times t3 and t6.

Case1: One other case of rumor spread at time t3. Nodes 1, 3, 4, 5, 6 and 7 are removal, node 2 is a lurker, and the other nodes are ignorant. In the end, the rumor will stop spreading, but there are still ignorants and lurkers in the network.

Case2: One other case of rumor spread at time t6. Node 11 is an ignorant, and others are removals. However, there are still ignorants in the network.

In the SEIR model, the state transition process of susceptible nodes is $S(Susceptible) \rightarrow E(Exposed) \rightarrow I(Infected)$, but in ILSR model, the state of an ignorant can be $I(Ignorant) \rightarrow L(Lurker)$, or $I(Ignorant) \rightarrow S(Spreader)$. Because in real life, people with weak ability to distinguish information are more likely to spread directly, there will be no lurk phase, and a strong ability to distinguish rumor information is more likely to be suspicious, become a lurker, and delay the dissemination of this information. Therefore, the population classification and node state transfer process of the SEIR model and the ILSR model are different.

Besides, we divide users by an average degree $\bar{k}$, which does not mean that 50% of users are important users, and 50% of users are ordinary users. Since the actual network is mostly sparse, so only a small number of nodes are divided into important nodes, and the proportion of important users and ordinary users divided by the ILSR model is closer to the 80/20 law (Pareto's principle, or the Bale Law), which is similar to the situation of fewer important nodes in social networks.

To sum up, compared with the mean-field approach to analysis rumor spreading, the ILSR model considers the differences of individuals and the influence of network topology on rumor spreading.

4. Experimental results and analysis

In this section, we simulate the spread of rumors in several different networks. These networks include regular network, WS network, BA scale-free network, and ego-Facebook network collected on Facebook social networks [27]. We obtained it from the Stanford large network dataset collection (http://snap.stanford.edu/data/).

Table 1
Networks information.

	Nodes	Edges	Density	Average clustering coefficient
Regular Network 1	1000	2 500	0.0050	0.0047
Regular Network 2	5000	20 000	0.0016	0.0015
WS Network 1	1000	2 000	0.0040	0.2993
WS Network 2	5000	15 000	0.0012	0.3750
BA scale-free Network 1	1000	1 996	0.0039	0.0344
BA scale-free Network 2	5000	89 676	0.0072	0.0285
ego-Facebook	4039	88 234	0.0108	0.6055

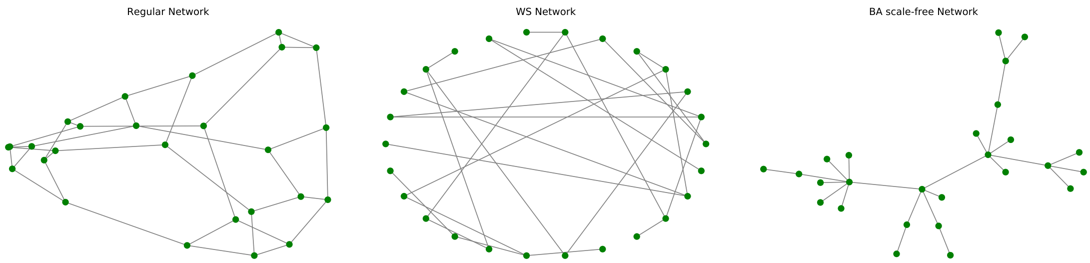

Fig. 7. Regular Network, WS network and BA scale-free Network of 26 nodes.

The information of several networks includes the number of nodes, the number of edges, the density of the network, and the average clustering coefficient. The information is shown in Table 1.

It can be seen from Table 1 that the number of nodes of the Regular Network 1, the WS Network 1 and the BA scale-free Network 1 is 1000, and there are differences in other information of the network because of the different network structures. The number of nodes of the Regular Network 2, the WS Network 2, and the BA scale-free Network 2 is 5000. Ego-Facebook, a social network from Facebook, has 4039 nodes. The structure of several networks is shown in Fig. 7.

In order to verify the correctness of our analysis results, rumor spreading simulation was performed in the above networks. The initial network $S(0) = 1$, $I(0) = N - S(0)$, $L(0) = 0$, $R(0) = 0$. The conversion rates α_1, α_2, β and δ of each node are dynamically calculated based on the network structure.

The experimental results are shown in Figs. 8, 9, and 10.

Fig. 8 shows the trend of the number of $I(t)$, $L(t)$, $S(t)$ and $R(t)$ of the ILSR model over time in Regular Network 1, WS Network 1, and BA scale-free Network 1. In Regular Network 1, because all nodes are of the same degree, it is impossible to distinguish between important nodes and ordinary nodes. In the whole spreading process, the number of lurkers $L(t) = 0$. The number of ignorants is decreasing slowly, and spreaders increases to the peak and then decreases. As can be seen from the partial enlargement, finally $I(t)$ and $S(t)$ are zero, all nodes become removals, and there is no rumor spreading in the network. In the WS Network 1, because the degree of nodes is different, the number of lurkers and spreaders has reached different peaks in the process of rumor propagation, and then gradually decreased. However, it can be seen from the partial enlarged detail that there are still ignorants and lurkers, indicating that the topology of the network has affected the results of the rumor spreading. In the BA scale-free Network 1, the results are similar to the WS Network 1, and there are still a few of ignorants and lurkers. It shows that compared with the existing analytical rumors on the homogeneous network, the ILSR model can effectively reflect the influence of individual differences and the actual network structure on the results of rumors.

Fig. 9 shows the trend of the number of $I(t)$, $L(t)$, $S(t)$ and $R(t)$ of the ILSR model over time in Regular Network 2, WS Network 2, and BA scale-free Network 2. In Regular Network 2, the maximum value of $S(t)$ is significantly increased, indicating that as the number of nodes increases, the range of rumors affects significantly. As can be seen from the partial enlargement, there are still some lurkers in the final results of the WS Network 2. In BA scale-free Network 2, due to the special network structure, the peak value of the propagator tends to 1.0, indicating that the rumor has the widest influence range. All the above results show that the topology of the network will affect the spread of rumors, which is consistent with the results of our analysis.

Fig. 10 shows the results of the rumor spread on ego-Facebook from the Facebook social network. As can be seen, compared with the previous network, the curve of the spreader has an obvious change in the upward trend, first increasing, then decreasing, and then sharply increasing to the maximum. When $t = 7$, the number of spreaders reaches the first peak, and then decreases, because the rumor spreading is affected by the structure of the network, which leads to a decrease in the rate of rumor propagation. As the propagation continues, the number of spreaders will eventually reach the maximum peak. Similarly, the number of lurkers also changes frequently. After several increases and decreases, there are still a few lurkers in the network. It also shows that the rumor propagation process in the real network is extremely

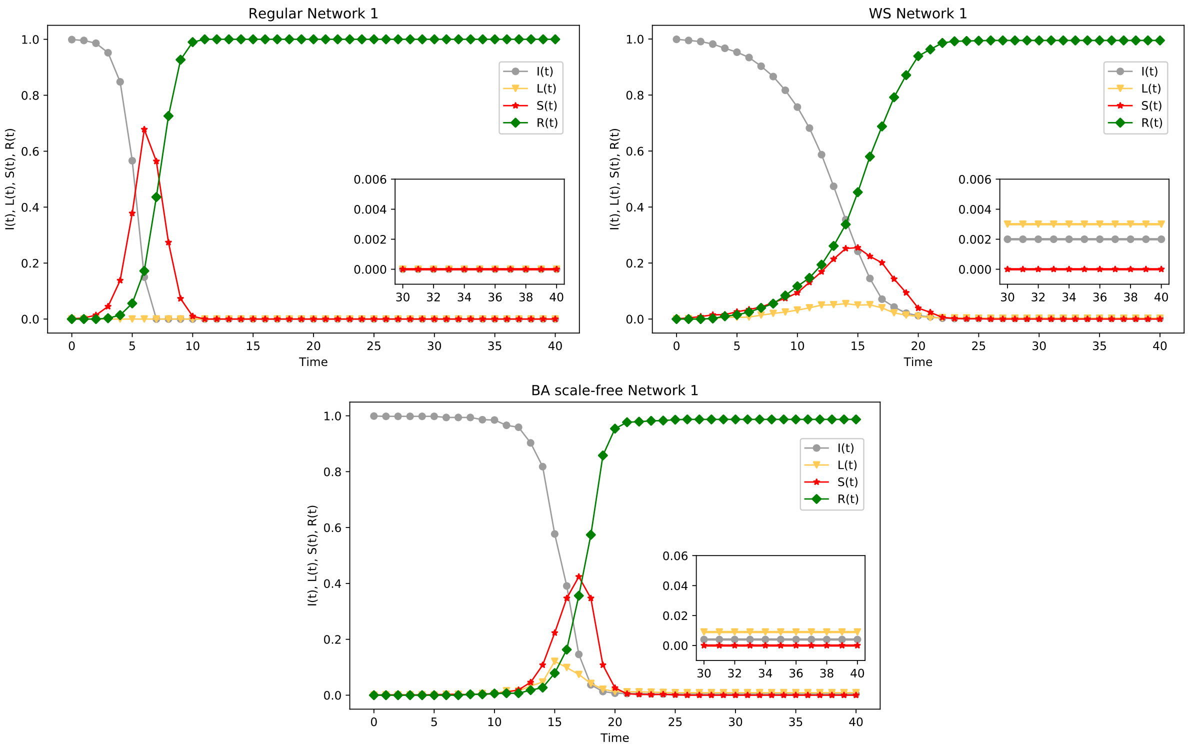

Fig. 8. Rumor spreading in Regular Network 1, WS network 1 and BA scale-free Network 1. $S(0) = 1$, $N = 1000$.

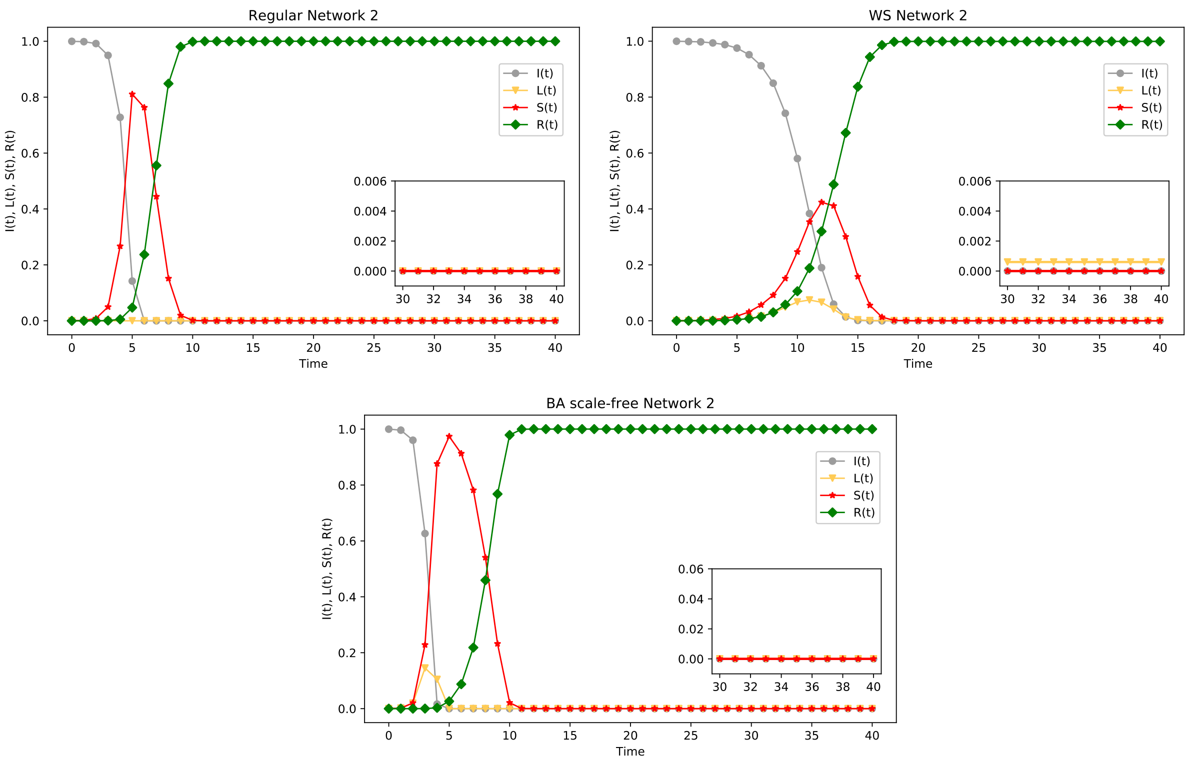

Fig. 9. Rumor spreading in Regular Network 2, WS network 2 and BA scale-free Network 2. $S(0) = 1$, $N = 5000$.

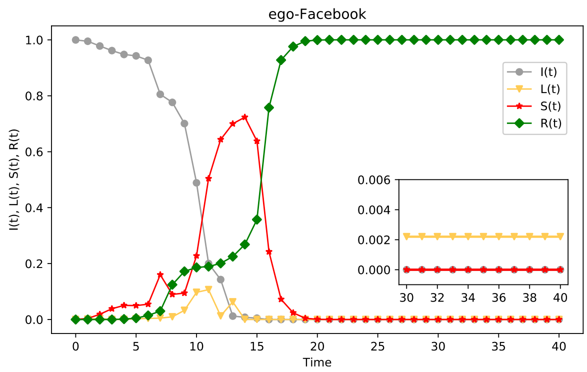

Fig. 10. Rumor spreading in ego-Facebook. $S(0) = 1$, $N = 4039$.

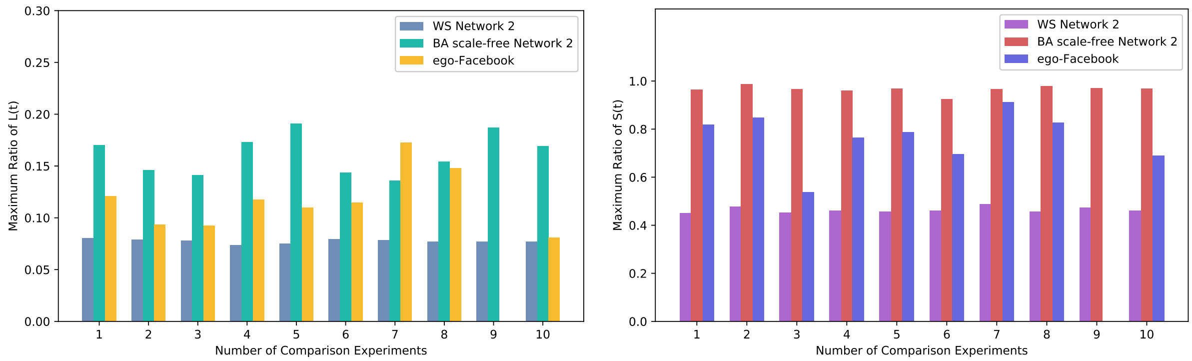

Fig. 11. The maximum ratio of lurkers and spreaders in different networks.

complex. The rumor analysis model based on the mean-field approach cannot depict such a complex propagation process, while the ILSR model can more realistically reflect the propagation law of rumors in the real world.

In order to further illustrate the changes in the number of lurkers and spreaders during the propagation process, several experiments were performed to record the maximum ratio of different $L(t)$ and $S(t)$ at the end of the propagation, as shown in Fig. 11.

Fig. 11 shows ten experiments in the WS Network 2, BA scale-free Network 2 and ego-Facebook network, and we recorded the maximum ratio of lurkers and spreaders. It can be seen that in the BA scale-free Network 2, the number of lurkers and spreaders is the highest, while in the WS Network 2 is the lowest. In the ego-Facebook Network, the number is between the first two. It can also be obtained that the initial values of the three networks are the same, the experimental results of several simulation rumor propagation are quite different. In WS Network 2, the maximum of $S(t)$ is relatively stable, ranging from 0.4 to 0.6, and, the maximum of $S(t)$ is close to 1.0 in BA scale-free Network 2. However, the maximum of $L(t)$ and $S(t)$ in ego-Facebook network is relatively vary widely. Considering the complexity of the real network topology, it is relatively difficult to predict the influence of rumor propagation.

In different networks, we tested the number of different initial spreaders and compared the influences of $S(0) = 1$, $S(0) = 10$ and $S(0) = 100$ on rumor propagation. The experimental results are shown in Fig. 12.

Fig. 12 shows the influence of different initial conditions on rumor propagation. It can be seen that when $S(0) = 100$, the time taken for $S(t)$ to reach the peak in different networks becomes shorter. When $S(0) = 1$, $S(t)$ took the longest time to reach the peak, especially in WS Network 2 and ego-Facebook network. In addition, there is no significant increase in the number of $L(t)$ and $S(t)$ in WS Network 2 and BA scale-free Network 2. However, in the ego-Facebook network, as the number of initial spreaders increases, the $S(t)$ of increases significantly, which indicates that in the actual network, the large-scale outbreak of rumors will accelerate the spread of rumors, and the number of people affected will also increase.

As shown in Fig. 13, the influence of initial rumors occurring on large nodes or small nodes on rumor propagation is analyzed.

Fig. 13 shows the trend of the number of $L(t)$ and $S(t)$ over time for the initial $S(0) = 10$ at nodes with large nodes and small nodes in WS Network 2, BA scale-free Network 2, and ego-Facebook network. Compared to nodes with small nodes, rumors occurring on nodes with large nodes, and the process of spreading accelerates, but the maximum peaks of lurkers and spreaders do not change significantly. Therefore, it is necessary to supervise and control the propagation state of the large nodes to achieve the purpose of slowing down and controlling the spread of rumors in real life.

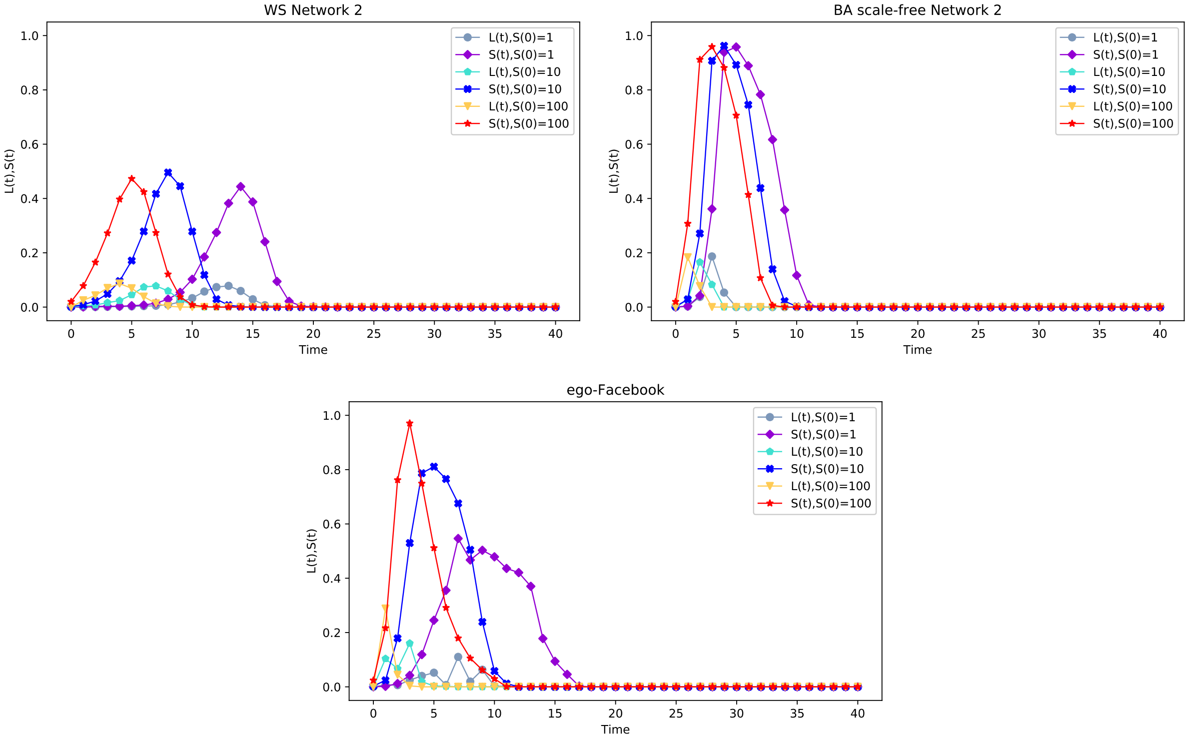

Fig. 12. Influence of different initial spreaders on rumor propagation.

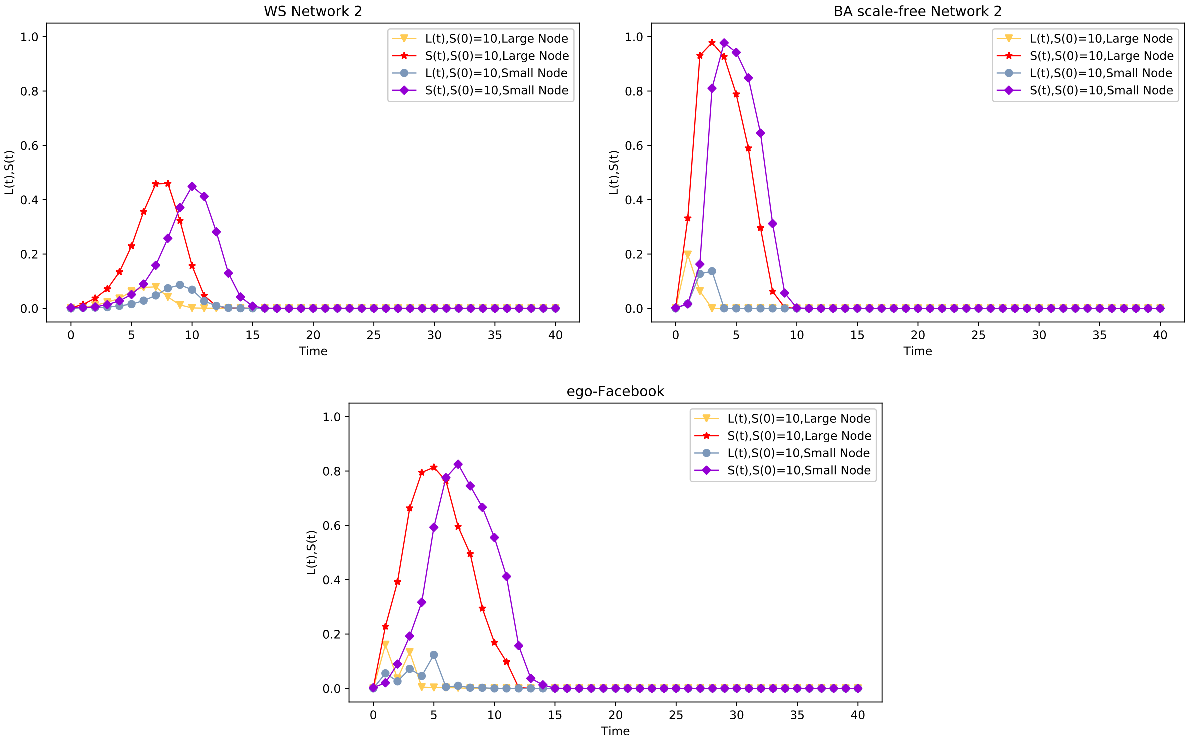

Fig. 13. The results of rumor occurring on large nodes and small nodes.

5. Conclusion

In this paper, in order to analyze the propagation law of rumors in complex networks, based on the existing research, we present a new ILSR rumor propagation model. Considering the different characteristics of different individuals' understanding and judgment of rumors, we divide users into important users and ordinary users, and combine them based on the network's degree. We calculated the equilibrium point of the model and the basic reproductive number and verified the correctness of our analysis results. Furthermore, we re-analyzed the model and designed a new state transition function based on the degree of different nodes to study the propagation law of rumors in complex networks. The model is validated by simulation experiments on the regular network, WS network, BA scale-free network, and a real network collected from Facebook. In different networks, we set up different comparative experiments, such as the influence of different numbers of Spreaders, the results of different degrees of nodes on rumor propagation, and analyzed the law of the change of different groups with time. The experimental results show that compared with the existing mean-field approach, the model can better describe the propagation process of rumors in complex networks. In order to understand the law of the spread of network rumors in the social network and study the factors affecting the process of the spread of rumors, it provides a reference for guiding and controlling the spread of network rumors.

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